## $k$-exact Reconstruction

- The problem of determining the point-wise variation of the flow variables from the cell averages is known as the reconstruction problem.
- Simply stated, the reconstruction problem involves the process of determining the point-wise distribution given the cell averages with the restriction that integrating the point-wise function recovers the cell averages.
- The variation of the flow quantities is approximated by a polynomial over each control volume.
- The polynomial satisfies a criteria called $k$-exactness which states that the polynomial reconstruction be exact i.e. recovers the cell average correctly whenever a polynomial of degree $k$ is used.
- A general polynomial of degree $k$ in 3D is written as

$$P^k(x,y,z) = \sum_{i=0}^{k} \sum_{j=0}^{k-i} \sum_{l=0}^{k-(i+j)} C_{i,j,l} x^i y^j z^l$$

  In 3D, the polynomial contains $\frac{(k+1)(k+2)(k+3)}{6}$ coefficients and in 1D and 2D, it contains $k+1$ and $\frac{(k+1)(k+2)}{2}$ coefficients respectively. e.g. $P^1(x,y,z) = C_0 + C_1 x + C_2 y + C_3 z$

- As a final task to complete the reconstruction, a support stencil of number equal to the number of coefficients is selected, and the resulting linear system solved. e.g for $P^1$ above

$$\begin{bmatrix} 1 & \overline{x}_1 & \overline{y}_1 & \overline{z}_1 \\ 1 & \overline{x}_2 & \overline{y}_2 & \overline{z}_2 \\ 1 & \overline{x}_3 & \overline{y}_3 & \overline{z}_3 \\ 1 & \overline{x}_4 & \overline{y}_4 & \overline{z}_4 \end{bmatrix} \begin{pmatrix} C_0 \\ C_1 \\ C_2 \\ C_3 \end{pmatrix} = \begin{pmatrix} \overline{Q}_1 \\ \overline{Q}_2 \\ \overline{Q}_3 \\ \overline{Q}_4 \end{pmatrix}$$