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# Direct Numerical Simulation (DNS) of Turbulent Flows

by
Anirudh Modi

Department of Aerospace Engineering
Penn State University

# Outline

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# Introduction

- Solve Navier-Stokes equation time-accurately without any modeling
  ie. exactly(ideal)/near-exact(practical)
  - possible mainly because of the advent of powerful supercomputers today
  - much more complex than RANS and LES

- Objective: not necessarily to reproduce real-life flows, but to perform controlled studies that allow better insight, scaling laws and turbulent models to develop (LES, RANS) [Hurdle : Re number]

- All length scales have to be resolved - leads to very fine grids => very time consuming
  - Captures turbulence and other nonlinear phenomena

# Introduction

- DNS is stressed as a “research tool” not as a brute force method. Reason : “near exact” solution as opposed to “exact”

- DNS => ideal => most complex
LES => next best => an order of magnitude less complex
RANS => coarsest => least complex (default, abundantly used)

# Background

- Foundation: Orszag & Patterson (1972)
  - Used spectral methods to perform $32^3$ computation of isotropic turbulence (Re=35)
- Rogallo (1981)
  - Used extension of Orszag-Patterson algorithm to compute homogeneous turbulence subjected to mean strain.
  - Compared to theory and experiments
  - Evaluated several turbulence models
  - Became the standard for DNS of homogeneous turbulence
- Kim et al (1987)
  - DNS of plane channel flow
- Spalart (1988)
  - Ingenious method to compute the turbulent flat-plate BL under zero and favorable pressure gradients

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# Background

- Pace of advancement has now increased
  - Le & Moin (1994) developed methods to specify inflow turbulence and hence computed reasonably complex flows like “flow over back-step”
  - Na & Moin (1996) computed flat plate BL separation
- However, Compressible turbulence is recent
  - Initiated by Feiereisen et al (1981)
  - Serious study started a decade later
  - Wall-bounded flows such as compressible channel and turbulent BL have only recently been attempted.
- Computation Aero-acoustics (CAA)
  - Exciting new development
  - Both fluid motion (large scale) and sound it radiates (small scale)are directly computed using DNS

# Background

- Main reason for progress:
  - Rapid progress in Computing Hardware
  - Currently available parallel machines like 64 processor SP-2 are 100 times faster than 64 processor ILLIAC-IV used in early 1980s

# Numerical Issues

- Spectral methods are most commonly employed
  - Approximates real-space function with series sum of orthogonal functions. Mathematically:

$$f(x_j) = \sum_{n=0}^{N-1} \hat{f}_n \phi_{n,j} \qquad j = \{0,1,\ldots,N-1\}$$

  - Fourier series for periodically assumed flows
  - Chebyshev polynomials for non-periodic flows
  - FFT which is O($N\log N$) instead of O($N^2$) makes it reasonably fast

# Spectral Methods



- Important observations on Spectral Methods
  - Extremely accurate and non-dissipative, enjoy exponential convergence
  - Orthogonal functions should be continuous, well behaved to reduce Gibb's phenomenon (to recover pointwise expntl accuracy)
  - Grid spacing - order of Kolmogorov scale of flow
  - Aliasing errors (false translation of new modes into domain) should be avoided - can cause numerical instability or excessive turbulence decay
  - Not clear how to extend this to curvilinear grids and hence complex geometry cannot be dealt with

# Finite Difference vs. Spectral

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- Rai & Moin (1991) compared the statistical results obtained by the two methods

- Concluded that spectral methods is most prevalent for turbulent flow DNS

- However, for complex geometries, high-order upward biased methods are very good

- Finite difference computations show reasonable but not excellent agreement with earlier results obtained by spectral methods

# Spatial Resolution

- Kolmogorov scale is most commonly quoted scale to be resolved

$$\eta = (\upsilon^3 / \varepsilon)^{1/4}$$

- This requirement is too stringent. Actually only $O(\eta)$ and not $\eta$. Depends on energy spectrum

- Mosir & Moin (1987) showed that most of dissipation in the curved channel occurs at scales greater than $15\eta$

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## Spatial Resolution

- Influenced by numerical method used (spectral methods are better)

- Differentiation error and errors due to nonlinearity of
- governing equations also affect

- Reynolds number is most important. DNS is restricted
- (by cost considerations) to low Re flows.

$$N_{DNS} = (0.088\,\mathrm{Re}_h)^{9/4}$$

- => Re of $10^6$ requires 133 billion grid points!!

- Optimal Re depends upon application. Re of DNS need not actually match real-life Re to be useful

# Temporal Resolution

- Wide range ot time scales makes system stiff for time advancement

- Implicit time advancement seems attractive as in CFD but large steps are not permitted => small scales can have large errors which corrupt solution

- Common practice in incompressible wall-bounded flows is to use implicit timestep for viscous terms and explicit timestep for convective terms

# Temporal Resolution



- For DNS of turbulent channel flow using implicit time-stepping, Choi & Moin (1994) showed that large time steps cause decay of turbulence to laminar state

- Reynolds number too plays important role in dictating the temporal resolution. Typically $N \sim Re^{3/4}$

# DNS and Experiments

- Results conistently show excellent comparison with experimental data

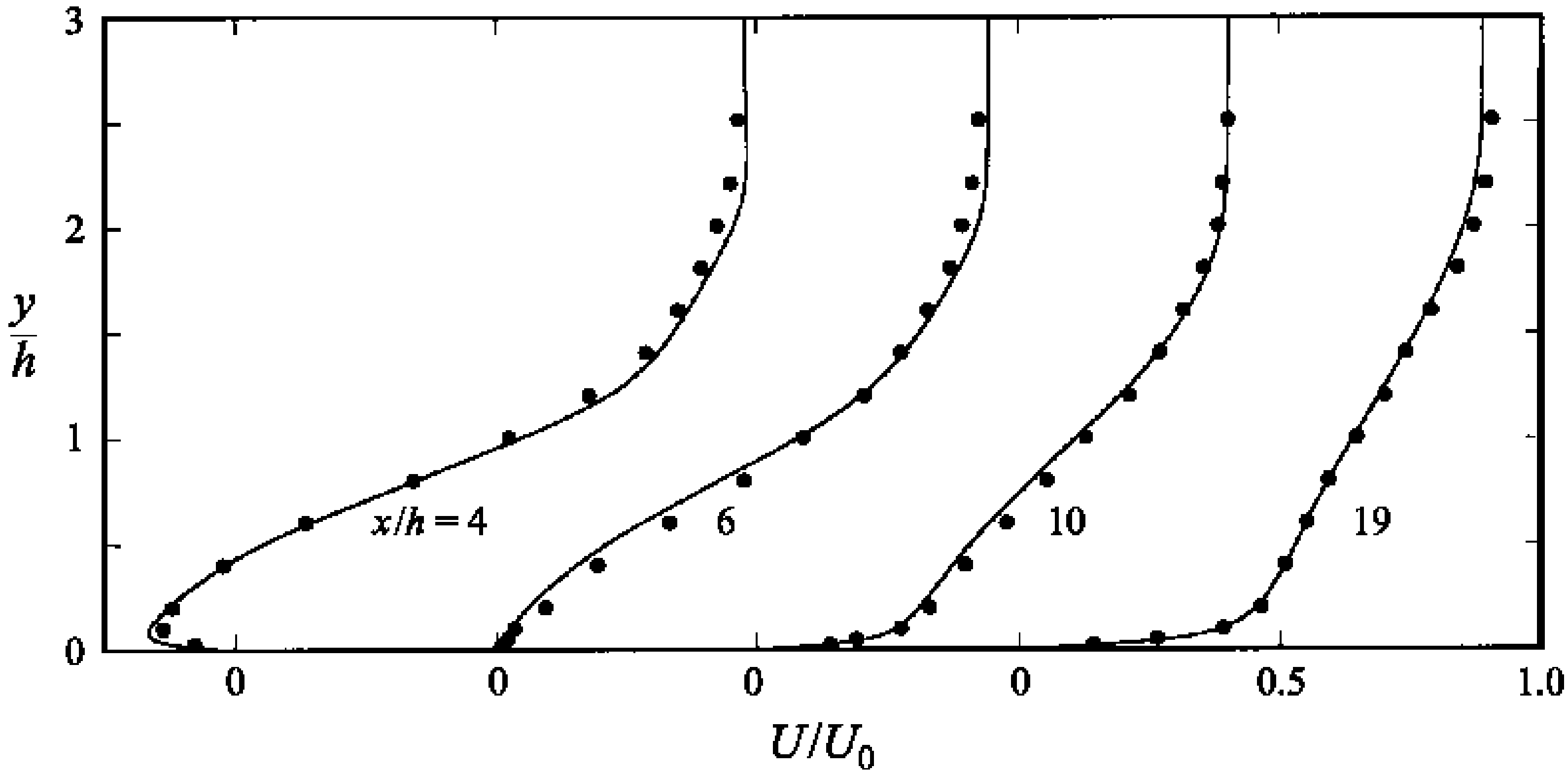
- Moin & Spalart (1987) used DNS data from a turbulent BL to estimate the accuracy of cross-wire probes and quantify the magnitudes of different sources of error

- DNS data has been recently used to provide probe design criteria for measurements of vorticity in turbulent flows

- Kim *et al*. (1987) performed DNS of turbulent channel (Re=3300) using about 4 million grid points and compared extensively with experimental data - found good agreement

# DNS and Experiments






Comparison of mean streamwise velocity profiles generated by DNS of turbulent flow over a backward-facing step (Le at al., 1997) with experimental data obtained by Jovic & Driver (1994)

# Conclusion

- Contributions of DNS to turbulence research has been impressive
- Future seems bright => faster computers
- Greatest advantage : stringent control it provides over the flow being studied
- Reynolds number still a bottleneck; however Re of simpler turbulent flows are currently approaching those of smaller scale experiments. DNS of forced isotropic turbulence has been conducted on $512^3$ grids by several people

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# Conclusion

- Databases generated by DNS provide results on turbulent flow statistics which are in good agreement with experiments => has greatly increased the confidence in DNS

- DNS data is extensively used to evaluate various
- LES models. It

- Availability of detailed flow information provided by DNS has increased understanding of physics.

- Very good correlation with experiments has made DNS synonymous with the term “Numerical experiment”